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NUMERICAL STUDY OF THE STRUCTURE OF A FINELY DISPERSE UNSTEADY TWO-PHASE JET

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The structure of an unsteady two-phase jet is numerically studied within the framework of the model of a heterogeneous medium with nonequilibrium velocities and temperatures with allowance for particle collisions and intergranular pressure.

Key words: *finely disperse two-phase jet, heterogeneous medium, intergranular pressure.*

Introduction. The study of two-phase jet flows with a high concentration of particles is very important for development of new technologies for fire extinction, displacement of massive bodies, washout of the soil layer and high-velocity drilling of channels in the layer, etc. To find the effective regime parameters, one has to know the laws of wave processes in two-phase media. These and associated issues were considered in [1–9].

The following scheme of exhaustion of an unsteady two-phase medium was examined most carefully. A cylindrical channel is initially filled by a high-pressure gas and densely packed particles. The high-pressure chamber is separated from the ambient medium by a membrane. When the membrane is removed and the initial discontinuity is destroyed, the joint wavy motion of the two-phase medium and ambient gas begins.

An exact self-similar solution for a one-dimensional rarefaction wave was found within the framework of the concept of a high-concentration effective gas [5, 8]. The behavior of an unsteady two-phase jet with coarsely disperse particles with allowance for particle collisions was numerically examined in [6]. The initial two-dimensional stage of discontinuity decay and propagation of the shock wave, the contact surface between the gas and the two-phase medium, and longitudinal and transverse rarefaction waves was considered in [8], where exact and approximate partial analytical solutions of unsteady (moving) wave structures in the ambient gas and in the exhausting two-phase medium were obtained. At the same time, a detailed analysis of experimental results shows that exhaustion of a gas with suspended particles in a wide range of initial pressures in the channel is associated with formation of quasi-steady barrel-shaped structures in characteristic flow regions. The flow with alternated compression and rarefaction regions is formed at subsonic (relative to the velocity of sound in the gas) velocities of gas-suspension exhaustion. In the experiments, the critical velocity at the channel exit reached 10 to 50% of the velocity of sound in the carrier gas. This phenomenon has not yet found a theoretical explanation.

The present work deals with studying the structure of two-phase disperse jets formed in accordance with the scheme described above and the influence of dispersion, effects of velocity and temperature nonequilibrium of the phases, and particle collisions on the flow character.

Formulation of the Problem. We consider a two-phase medium under known assumptions [10] with allowance for inertial effects in the flow around the particles, random motion of the disperse phase, and its intergranular pressure [6, 7, 11]. We assume that the principle of identical distribution of energy of random motion of colliding "rough" disperse particles over the degrees of freedom is valid. We also neglect the fluctuating turbulent energy of the carrier gas, as compared to the energy of random motion of particles for $\rho_1^\circ/\rho_2^\circ \ll 1$ (this issue was discussed in [11]).

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The problem in the formulation considered is described as follows:

$$
\begin{gathered}
\frac{\partial \rho_i}{\partial t} + \nabla \cdot \rho_i \boldsymbol{v}_i = 0, \\
\frac{\partial \rho_1 \boldsymbol{v}_1}{\partial t} + \nabla \cdot \rho_1(\boldsymbol{v}_1\boldsymbol{v}_1) + \beta_1 \nabla p + (1-\beta_2)\nabla p_{2*} = -\beta_3 \boldsymbol{F}_\mu + \beta_3\rho_1\boldsymbol{g} + (1-\beta_2)(\rho_1+\rho_2)\boldsymbol{g}, \\
\frac{\partial \rho_2 \boldsymbol{v}_2}{\partial t} + \nabla \cdot \rho_2(\boldsymbol{v}_2\boldsymbol{v}_2) + (1-\beta_1)\nabla p + \beta_2\nabla p_{2*} = \beta_3 \boldsymbol{F}_\mu - \beta_3\rho_1\boldsymbol{g} + \beta_2(\rho_1+\rho_2)\boldsymbol{g}, \\
\frac{\partial \rho_2 u_2}{\partial t} + \nabla \cdot \rho_2 u_2 \boldsymbol{v}_2 = Q + H_{sh}, \\
\frac{\partial \rho_2 k_2}{\partial t} + \nabla \cdot \rho_2 k_2 \boldsymbol{v}_2 + p_{2*}\nabla \cdot \boldsymbol{v}_2 = H_M + \tilde{H}_\mu - H_\omega - H_{sh} - H_\mu, \\
\frac{\partial}{\partial t}(\rho_1 E_1 + \rho_2 E_2) + \nabla \cdot [\rho_1 E_1 \boldsymbol{v}_1 + \rho_2 E_2 \boldsymbol{v}_2 + p(\alpha_1\boldsymbol{v}_1 + \alpha_2\boldsymbol{v}_2) + p_{2*}\boldsymbol{v}_2] = \rho_1\boldsymbol{g}\cdot\boldsymbol{v}_1 + \rho_2\boldsymbol{g}\cdot\boldsymbol{v}_2, \\
\rho_i = \rho_i^\circ \alpha_i \quad (i=1,2), \qquad E_1 = u_1 + v_1^2/2, \qquad E_2 = u_2 + k_2 + v_2^2/2, \\
\beta_1 = \frac{\alpha_1(2+\chi_m\rho_1^\circ/\rho_2^\circ)}{2+\chi_m(\alpha_2+\alpha_1\rho_1^\circ/\rho_2^\circ)}, \quad \beta_2 = \frac{2+\chi_m\alpha_2}{2+\chi_m(\alpha_2+\alpha_1\rho_1^\circ/\rho_2^\circ)}, \quad \beta_3 = \frac{2}{2+\chi_m(\alpha_2+\alpha_1\rho_1^\circ/\rho_2^\circ)}.
\end{gathered}
\tag{1}
$$

Hereinafter, the subscripts 1 and 2 refer to parameters of the carrier and disperse phases, respectively, the circle at the top indicates the true densities, and ∇ is the Hamiltonian operator. The quantities α_i, ρ_i, $\boldsymbol{v}_i$, E_i, u_i, p, p_{2*}, k_2, and $\boldsymbol{g}$ are the volume fraction, reduced density, velocity vector, total and internal energies of a unit mass of the ith phase, gas pressure, effective pressure caused by random motion of particles, fluctuating energy of a unit mass of the disperse phase, and the vector of acceleration of the gravity force, $\boldsymbol{F}_\mu$, Q, H_M, $\tilde{H}_\mu$, H_ω, H_μ, and H_{sh} are the viscous component of the interphase interaction force, power of heat transfer between the gas and particles, powers of generation of energy of random motion of particles due to the Magnus force and the vortex flow around the particles, dissipation because of random rotation, translational motion of particles, and inelastic collisions, χ_m is a coefficient taking into account the effect of nonuniqueness and a nonspherical shape of particles on the added mass force ($\chi_m = 1$ for spherical particles), and t is the time.

To close system (1), we use the equation of state of a calorically perfect ideal gas

$$p = (\gamma_1 - 1)\rho_1^\circ u_1, \qquad u_1 = c_v T_1$$

and the equation of state of incompressible solid particles

$$u_2 = c_2 T_2, \qquad \{\gamma_1, c_v, c_2, \rho_2^\circ\} \equiv \text{const.}$$

Here T_1 and T_2 are the temperatures of the carrier phase and particles, γ_1 and c_v are the ratio of specific heats and specific heat capacity of the gas at constant volume, and c_2 is the specific heat capacity of particles.

The intensities of interphase friction $\boldsymbol{F}_\mu$ and heat transfer Q are set on the basis of the known empirical relations tested for the class of problems considered [10, 12, 13]. The power of generation of energy of random motion of particles owing to the action of transverse Magnus forces on rotating particles H_M is set on the basis of Goldshtik's model [11]. To describe the intensity of generation of random motion of particles $\tilde{H}_\mu$ due to fluctuations of the longitudinal and transverse forces because of the vortex flow around particles at Reynolds numbers $\mathrm{Re}_{12} > 10^2$, we use the results of [6, 7]. We take into account the dissipative mechanisms of transformation of energy of random motion of particles H_ω and random translational motion of particles H_μ to internal energy of the gas, and also dissipation of kinetic energy of random motion of particles in collisions H_{sh} in the form [7, 11].

The problem of exhaustion of a two-phase disperse medium from a cylindrical channel into the atmosphere was solved with the following initial data: length of the high-pressure chamber $L = 2$ m, channel diameter $D = 0.2$ m, $p_\mathrm{h} = 5$ MPa, $p_\mathrm{a} = 0.1$ MPa, $T_{i,h} = T_{i,a} = 293$ K, $\alpha_{1\mathrm{h}} = 0.4$, $\alpha_{1\mathrm{a}} = 1$, $\gamma_1 = 1.4$, $\mu_1 = 1.8 \cdot 10^{-5}$ Pa $\cdot$ sec, $\lambda_1 = 0.025$ W/(m $\cdot$ K), $R_1 = 287$ J/(kg $\cdot$ K), $c_v = 716$ m^2/(sec$^2 \cdot$ K), $\rho_2^\circ = 1500$ kg/m^3, and $c_2 = 710$ m^2/(sec$^2 \cdot$ K), where R_1 is the gas constant; the subscripts h and a refer to parameters in the high-pressure chamber and ambient parameters, respectively.

TABLE 1

d, μm	Sh	u_1	u_2	p	α_2
5	0.5	0.546	0.545	0.250	0.724
	1.0	0.548	0.547	0.250	0.725
	1.5	0.548	0.547	0.249	0.725
	2.0	0.546	0.545	0.248	0.726
	2.5	0.456	0.456	0.193	0.772
25	0.5	0.561	0.542	0.247	0.722
	1.0	0.557	0.545	0.247	0.723
	1.5	0.555	0.546	0.248	0.724
	2.0	0.545	0.539	0.243	0.727
	2.5	0.458	0.455	0.191	0.772
50	0.5	0.573	0.540	0.245	0.721
	1.0	0.566	0.544	0.246	0.722
	1.5	0.562	0.545	0.247	0.723
	2.0	0.546	0.534	0.239	0.729
	2.5	0.461	0.454	0.189	0.772
100	0.5	0.587	0.538	0.243	0.719
	1.0	0.576	0.542	0.245	0.721
	1.5	0.571	0.544	0.245	0.722
	2.0	0.548	0.529	0.234	0.730
	2.5	0.465	0.451	0.186	0.772

The problem posed is described by a system of stiff equations containing the "fast" and "slow" components of the solution [14, 15]. Indeed, the characteristic times of interphase velocity relaxation of the phases differ by more than an order of magnitude:

$$\frac{t_1^{(v)}}{t_2^{(v)}} = \frac{t_1^{(\mu)}}{t_2^{(\mu)}} = \frac{\rho_{1\mathrm{h}}^\circ \alpha_{1\mathrm{h}}}{\rho_{2\mathrm{h}}^\circ \alpha_{2\mathrm{h}}} = 0.027.$$

Therefore, we used a K-stable difference scheme for numerical simulations of a finely disperse unsteady jet. The calculations were performed in a cylindrical coordinate system with axial symmetry on a 600×100 grid. The boundary conditions of the problem were set as follows: no-slip conditions on the wall and extrapolation of parameters at the boundaries of the computational domain.

Results of Numerical Simulations. After decay of the initial discontinuity, a rarefaction wave propagates from the channel exit to its bottom, and jet exhaustion of the two-phase medium in the opposite direction begins. We introduce the characteristic time of the process (Strouhal number) based on the channel depth and effective velocity of sound: $\mathrm{Sh} = a_\mathrm{h} t/L$.

In the critical cross section, in the range $0 < \mathrm{Sh} < 2$ (time of propagation of the rarefaction wave to the channel bottom and its return to the discontinuity-decay point), the exact analytical dimensionless values of parameters are constant [5, 8]:

$$\mathrm{M}_* = 0.548, \qquad p'_* = 0.249, \qquad \alpha_{1*} = 0.725, \qquad \rho'_* = 0.458 \tag{2}$$

($\mathrm{M} = v/a_\mathrm{h}$ is the Mach number, $p' = p/p_\mathrm{h}$, and $\rho' = \rho/\rho_\mathrm{h}$).

This self-similar solution is the limit as $d \to 0$ (d is the particle diameter) to which the solution converges within the framework of the model of the two-velocity two-temperature medium (1). For $d > 5$ μm, the differences in velocities and temperatures of the phases and the effects of interphase interaction become more pronounced (see Table 1); as was found earlier [5], a quasi-critical flow of the two-phase medium with characteristic velocities of the gas and disperse phases is observed. Thus, in the considered range of initial parameters with $0 < \mathrm{Sh} < 2$, the parameters of the mixture flow in the critical cross section, which does not coincide with the channel exit in the general case, are comparatively stable. Hence, a quasi-steady (steady at high times and constant critical parameters) exhaustion of the gas with suspended particles is formed in the vicinity of the channel exit in the indicated time interval.

The jet formed by exhaustion of the two-phase disperse medium from the cylindrical channel with an excess pressure into the atmosphere is underexpanded, i.e., the pressure at the exit is higher than the ambient pressure. Three variants of calculations for three values of the particle diameter d (5, 25, and 50 μm) are considered below.

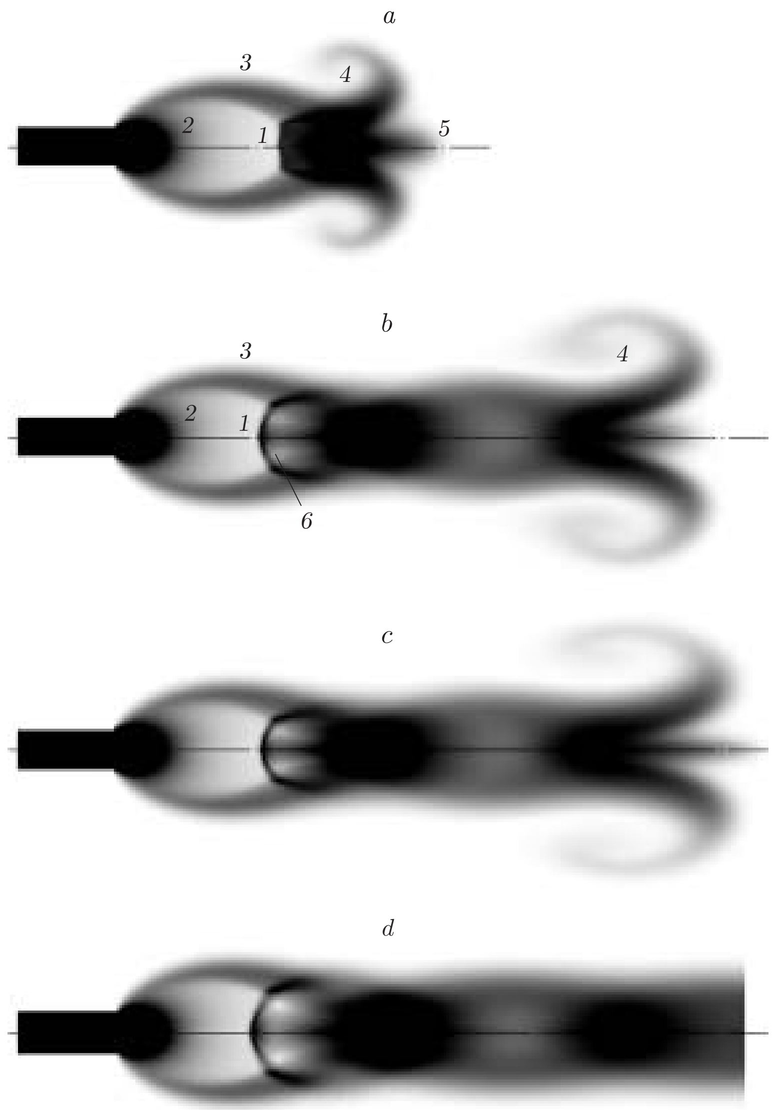

Fig. 1. Field of particle concentration for $d = 5$ μm: $\rho_2^\circ = 1500$ (a, b, and d) and 3000 kg/m^3(c); Sh = 1 (a) and 2 (b and c); steady flow (d).

The first variant corresponds to a finely disperse two-phase medium ($d = 5$ μm). A detailed analysis of calculation results shows that the interphase exchange processes are rather intense. Phase slipping is insignificant, and the medium as a whole behaves as an effective gas. Figures 1a and 1b show the fields of disperse phase concentration α_2 for Sh = 1 and 2, respectively. Hereinafter, the dark and light regions in the figure refer to higher and lower concentrations of particles, respectively. The calculation results obtained by model (1) with allowance for random motion of particles and the action of intergranular pressure of the pseudogas of the disperse phase are shown above the axis of symmetry, and the calculation results within the framework of the model of a collision-free disperse medium are plotted below the axis.

A typical feature of the numerical solution is substantial nonuniformity of density and pressure both along the jet and over the jet radius. These figures display the first "barrel" finalized by the Mach disk 1 with a subsonic velocity of particles (with respect to the effective velocity of sound of the mixture) behind the Mach disk. The curvilinear barrel shock 2 and the reflected shock 3 are adjacent to the barrel. The reverse vortex motion of the gas and particles (region 6) is observed behind the Mach disk. In the head part of the jet, there is a vortex region 4.

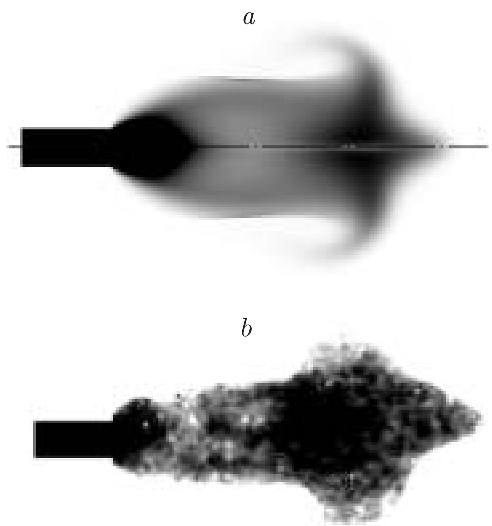

Fig. 2. Field of particle concentration for $d = 25$ μm and Sh $= 1$: calculations (a) and experiment (b).

A cumulative effect (region 5 in Fig. 1a) is observed for an almost uniform distribution of flow parameters at the channel exit near the axis of symmetry (at a distance approximately equal to $15D$). It is important to note that a nonuniform (non-isobaric) flow field is observed only in the region occupied by the disperse phase. The supersonic (with respect to the effective velocity of sound) two-phase dense jet is subsonic with respect to the velocity of sound in the carrier gas. For instance, the maximum velocity of the gas and particles in the jet at the time corresponding to Sh $= 1$ is approximately 150 m/sec. Obviously, no shock-wave structures arise in "pure" gas flows with such parameters.

To estimate the influence of the granular material density, we also calculated the exhaustion of a gas-disperse medium with particles of diameter $d = 5$ μm and twofold true density ($\rho_2^\circ = 3000$ kg/m^3). The field of particle concentration for Sh $= 2$ is shown in Fig. 1c. A comparison with Fig. 1b shows that the numerical solution is self-similar in terms of the varied parameter of true density. This result agrees with the partial analytical solutions [8], which are independent of density of the disperse phase material.

Figure 1d shows the steady flow field of the gas with suspended particles for $d = 5$ μm and $\rho_2^\circ = 1500$ kg/m^3, which was obtained numerically by the pseudo-transient method with constant critical parameters (2) in time and over the radius at the channel exit. A comparison with previous calculations testifies to formation of quasi-steady shock-wave structures in the considered problem of exhaustion of a two-phase medium from a bounded cylindrical channel.

The field of particle concentration calculated for $d = 25$ μm and Sh $= 1$ is shown in Fig. 2a. As in Fig. 1, the flow field of the two-phase medium that takes into account particle interaction is shown above the axis of symmetry and the flow field with particle collisions ignored is shown below the axis. The nonuniformity of jet parameters is retained, but the flow structure is smeared. The reason is that the effects of phase slipping, differences in thermodynamic parameters, and random motion of particles become more pronounced.

An experiment was performed with the same conditions as those in calculations. Sieved silica sand with particle-material density $\rho_2^\circ = 2600$ kg/m^3 was used as a disperse medium. Figure 2b shows the photograph of the visible contour of the jet. The asymmetric shape of the jet and the asymmetric distribution of sand-particle density in the photograph are caused by the experimental error, in particular, with non-instantaneous breakdown of the membrane separating the high-pressure gas mixture and sand of bulk density in the channel from the ambient medium. The character of exhaustion of the two-phase medium registered in the experiment is in qualitative agreement with the calculation results (Fig. 2a).

With a further increase in particle diameter up to $d = 50$ μm and higher, the structure of the two-phase disperse jet is not revealed, which is explained by expansion of relaxation regions.

Conclusions. 1. A cumulative effect in the head part of a two-phase jet was found in the case of uniform parameters of the flow exhausting from a cylindrical channel.

2. Formation of quasi-steady (or steady in the limit) two-dimensional wave structures in the jet flow of the gas with finely disperse suspended particles of self-similar granules in terms of material density was observed, the non-isobaric two-phase jet and shock-wave structures being formed in the subsonic (with respect to the gas velocity) flow regime.

3. The influence of the particle density and diameter, differences in velocities and temperatures of the phases, random motion of particles, and intergranular pressure on the character of the gas-suspension flow was analyzed.

4. A relationship between the dispersion of the exhausting two-phase disperse jet and manifestation of its structural properties was established.

5. The results of numerical simulations are in agreement with experimental data.

REFERENCES

1. E. V. Vorozhtsov, A. V. Fedorov, and V. M. Fomin, "On the theory of the flow of a mixture of gas and coal particles with allowance for desorption," in: *Aeromechanics* [in Russian], Nauka, Moscow (1976), pp. 316–327.
2. A. V. Fedorov, "Analysis of equations that describe sudden outburst of coal and gas," in: *Numerical Methods of Mechanics of Continuous Media* (collected scientific papers) [in Russian], Vol. 11, No. 4 (1980), pp. 145–153.
3. Yu. V. Kazakov, A. V. Fedorov, and V. M. Fomin, "Calculation of the dispersion of a compressed volume of a gas suspension," *J. Appl. Mech. Tech. Phys.*, **28**, No. 5, 773–777 (1987).
4. S. D. Lyubarskii and A. S. Ivanov, "Motion of a compressed two-phase medium of bulK density upon sudden expansion," *Combust. Expl. Shock Waves*, **25**, No. 3, 335–337 (1989).
5. A. S. Ivanov, V. V. Kozlov, and D. V. Sadin, "Unsteady exhaustion of a two-phase disperse medium from a finite-size cylindrical channel into the atmosphere," *Izv. Ross. Akad. Nauk, Mekh. Zhidk. Gaza*, No. 3, 60–66 (1996).
6. D. V. Sadin, "Behavior of the unsteady jet of a mixture of a pressurized gas and dispersed particles discharged from a circular duct into the atmosphere," *J. Appl. Mech. Tech. Phys.*, **40**, No. 1, 130–135 (1999).
7. D. V. Sadin, "Calculation of the acceleration of a cylindrical body by an unsteady two-phase flow," *Combust. Expl. Shock Waves*, **35**, No. 5, 591–597 (1999).
8. D. V. Sadin, "Pulsed discharge of a two-phase medium from a bounded duct capable of longitudinal motion," *J. Appl. Mech. Tech. Phys.*, **41**, No. 1, 76–83 (2000).
9. D. V. Sadin and K. N. Egorov, "Interaction of an unsteady two-phase jet with a layer of a disperse medium," *J. Appl. Mech. Tech. Phys.*, **42**, No. 5, 827–832 (2001).
10. R. I. Nigmatulin, *Dynamics of Multiphase Media*, Part 1, Hemisphere Publ., New York (1991).
11. R. I. Nigmatulin, *Fundamentals of Mechanics of Heterogeneous Media* [in Russian], Nauka, Moscow (1978).
12. L. E. Sternin, B. P. Maslov, A. A. Shraiber, and A. M. Podvysotskii, *Two-Phase Mono- and Polydispersed Flows of Gas and Particles* [in Russian], Mashinostroenie, Moscow (1980).
13. A. F. Chudnovskii, *Heat Transfer in Disperse Media* [in Russian], Gostekhteorizdat, Moscow (1954).
14. D. V. Sadin, "Stiffness problem in modeling wave flows of heterogeneous media with a three-temperature scheme of interphase heat and mass transfer," *J. Appl. Mech. Tech. Phys.*, **43**, No. 2, 286–290 (2002).
15. D. V. Sadin, "Stiffness of systems with partial derivatives that describe motion of heterogeneous media," *Mat. Model.*, **14**, No. 11, 43–53 (2002).